Cube cyl.

Conditions pour une courbe donnée par son repère intérieur soit cylindrique.

$\rho = \rho(t)$

$\tau = \tau(t)$

$\boxed{\frac{dt}{ds} \text{ arbitraire.}}$

$$\frac{dt}{ds} = f(t)$$

revient à satisfaire $\frac{d\psi}{ds} = \sin\varphi\,\rho$ $\qquad \psi' f = \sin\varphi\,\rho$

$$\tau + \frac{d\rho}{d\psi}\cos\varphi = \frac{3xy}{a} \qquad \tau + \frac{d\rho}{d\psi}\cos\varphi = \frac{3xy}{a}$$

$$\left(\tau - 3\frac{xy}{a}\right)^2 d\psi^2 = d\rho^2\left(1 - \psi'^2 f^2\right)$$

On a toujours $\lfloor a\rho c = x^2$ $\qquad \tau + \frac{d\rho}{d\psi}\cos\varphi = \frac{3xy}{a}$

$$(X-A)^2 - \left[(X-A)_u\right]^2 = a^2$$

Si cyl, $\psi' = \sin\varphi\,\rho$

059776

$$\tau + \frac{d\rho}{d\psi}\frac{x^2}{a\rho} = \frac{3xy}{a}$$

$$(a\tau - 3xy)\rho\,d\psi + x^2\rho' = 0$$

$$(a\tau - 3xy)\rho^2\sin\varphi + x^2\rho' = 0 \qquad \frac{x^4\rho'^2}{\rho^4(a\tau-3xy)^2} + \frac{x^4}{a^2\rho^2} = 1$$

$$a\rho c = x^2$$

$$c = \frac{x^2}{a\rho} = \frac{(3xy - a\tau)\psi'}{a\rho'}$$

$$\delta = \frac{\psi'}{\rho} \qquad \operatorname{tg}\varphi = \frac{\psi'}{\rho}\frac{a\rho'}{(3xy-a\tau)\psi'} = \frac{a\rho'}{\rho(3xy-a\tau)}$$

$$\frac{\varphi'}{\cos^2} = \frac{a\rho''}{\rho(\ )} - \frac{a\rho'\left[\rho'(3xy-a\tau) + a\rho\tau' + \rho\psi'(3y^2-3x^2)\right]}{\rho^2(3xy-a\tau)}$$

$$-\frac{\psi'}{\rho}\varphi' = \frac{2xy\psi'}{a\rho} - \frac{x^2\rho'}{a\rho^2}$$

$$c = \frac{x^2}{a\varrho} = \frac{(3xy - a\tau)\psi'}{a\varrho'}$$

$$s = \frac{\psi'}{\varrho} \qquad \operatorname{tg}\varphi = \frac{\psi'}{\varrho}\frac{a\varrho'}{(3xy-a\tau)\psi'} = \frac{a\varrho'}{\varrho(3xy-a\tau)}$$

$$\frac{\varphi'}{\cos^2} = \frac{a\varrho''}{\varrho(\ )} - \frac{a\varrho'\left[\varrho'(3xy-a\tau) + a\varrho\tau' + \varrho\psi'(3y^2-3x^2)\right]}{\varrho^2(3xy-a\tau)}$$

$$-\frac{\psi'}{\varrho}\varphi' = \frac{2xy\psi'}{a\varrho} - \frac{x^2\varrho'}{a\varrho^2}$$

$$c = \frac{(3xy-a\tau)s\varrho}{a\varrho'} = \frac{x^2}{a\varrho} \qquad a\varrho c = x^2$$

$$a\varrho' c + a\varrho s\varphi' = 2xy\,s\varrho \quad \Big|\ -3$$

$$s = \frac{x^2 a\varrho'}{a\varrho^2(3xy-a\tau)} \qquad a\varrho' c + a\tau\varrho s = 3xy\,s\varrho \quad \Big|\ 2$$

$$xy = \frac{a\varrho' c + a\tau\varrho s}{3\varrho s} \qquad -a\varrho' c + 2a\tau\varrho s + 3a\varrho s\varphi' = 0$$

$$a\varrho' c + a\tau\varrho s$$

$$-a\varrho' c^3 + 3a\varrho s c^2\varphi' + 2a\tau\varrho s c^2 = 0$$

$$(-a\varrho c^3)' + 2a\tau\varrho s c^2 = 0$$

$$c = \frac{x^2}{a\varrho} \qquad \left[\begin{array}{l} a\varrho c = x^2 \\ a\varrho' c + a\tau\varrho s = 3s\varrho xy \\ s\varrho = \psi' \end{array}\right. \qquad \frac{x^4}{a^2\varrho^2} + \frac{\psi'^2}{\varrho^2} = 1$$

$$-\frac{(a\varrho c^3)'}{a\varrho c^3} + 2a\tau\frac{s}{c} = 0$$

$$\tau = f^0$$

$$\frac{\varrho'}{\varrho}x^2 + a\tau\psi' = 3\psi' xy$$

$$\frac{s}{c} = +\frac{(a\varrho c^3)'}{a\varrho c^3}\cdot\frac{1}{2a\tau}$$

$$d\varrho x^2 + \varrho\psi'(a\tau - 3xy) = 0$$

$$(a c\varrho)(1 - a c\varrho) = \frac{a^2(\varrho' c + \tau\varrho s)^2}{9\varrho^3 s^2 c}$$

$$9\varrho^3 s^2 c = a\left[4\varrho^4 s^2 c^2 + (\varrho' c - \tau\varrho s)^2\right]$$

$$\frac{xy}{x} \quad \frac{3a\varrho^2 c s}{a\varrho' c + a\tau\varrho s} = \operatorname{tg}\psi$$

$$\varphi' = \frac{\varrho' c - 2\varrho\tau s}{3\varrho s}$$

$$\varphi'\left[(\varrho' c + a\tau\varrho s)^2 + 9\varrho^4 c^2 s^2\right] = (\varrho' c + \varrho\tau s)\left[6\varrho\varrho' c s + 3\varrho^2(c^2 - s^2)\varphi'\right] -$$

$$- 3\varrho^2 c s\left[\varrho'' c + \varrho'\tau s - \varrho\tau' s + (\varrho' s + \varrho\tau c)\varphi'\right]$$

$$\varrho s\left[(\varrho' c + \varrho\tau s)^2 + 9\varrho^4 c^2 s^2\right] = (\varrho' c + \varrho\tau s)\left[6\varrho\varrho' c s + 3\varrho^2(c^2 - s^2)\frac{\varrho' c - 2\varrho\tau s}{3\varrho s}\right] - 3\varrho^2 c s\left[\varrho'' c + \varrho' c s - \varrho\tau' s + \frac{(-\varrho s - \varrho\tau c)(\varrho' c - 2\varrho\tau s)}{3\varrho s}\right]$$

$p \to 0 \qquad z \to -\frac{p^2}{3}$

$s \sim p \qquad c = 1$

$$\alpha = -\frac{9s^3 c x^2 (y^2-x^2)^2}{2a^2y^2} \frac{4a^2y^2}{9x^2c^4(p^2+2y^2)^2} = \frac{2p^3(y^2-x^2)^2}{(p^2+2y^2)^2}$$

$$s' \sim \frac{3p^3 x s}{2a}\left(-\frac{p^2}{3}\right) \sim p^5$$

$$\tau = \frac{3x}{2ay}\left[s^2+2y^2c^2\right] = \frac{3xc^2(p^2+2y^2)}{2ay}$$

$$c_{ind} = \frac{3p^3x(y^2-x^2)}{2axy\,3xc^2(p^2+2y^2)} = \frac{p^3(y^2-x^2)}{p^2+2y^2}$$

$$\tau' = -\frac{9s^3cx^2(y^2-x^2)^2}{2a^2y^2}$$

$$\beta = \frac{3p^4x^2(2p^2+3c^2)(y^2-x^2)^2}{4a^2x^2} \frac{4a^2y^2}{9x^2c^4(p^2+2y^2)^2} = \frac{p^4(2p^2+3)(y^2-x^2)^2}{(p^2+2y^2)^2}$$

$$s' = \frac{3s^3xs}{2acy}(y^2-x^2)$$

$$\frac{ss''-2s'^2}{s^2} = \frac{27s^4x^2}{4a^2y^2}(y^2-x^2)^2$$

$$d = \frac{3x^2}{4c}\frac{2xy}{3xc^2(p^2+2y^2)} = \frac{2xy}{c^3(p^2+2y^2)}$$

$$\frac{s''}{s} = \frac{9s^6x^2}{2a^2c^2y^2}(y^2-x^2)^2 +$$

$$p^2+3z+3z^2=0$$

$$= \frac{9s^4x^2(2s^2+3c^2)}{4a^2c^2y^2}(y^2-x^2)^2$$

$$\psi' = -2\psi'\frac{cy}{sx} + \frac{3}{2}\frac{s^2x}{ay}(y^2-x^2)$$

$$= \frac{z'c}{2sx^2} + \frac{3}{2}\frac{sx^2}{ay}(y^2-x^2)$$

$z = y^2 - x^2 \qquad \psi' = -\frac{z'}{4xy}$

$z' = -4xy\psi'$

$$-\frac{2a^2\tau''}{y} = \frac{y^3\left[\ \right]}{4y^4}$$

$$2pp' + 3z' + 3zz' = 0$$

$$= \frac{2zx^2\left[3s^2c^2-s^4\right]\left(\frac{z'c}{2sx^2}+\frac{3s^2x}{2ay}z\right)}{y^2} + s^3c\,\frac{y^2\left[-z^2z'+2x^2zz'\right]-x^2z^2(\ldots)}{y^4} =$$

$$\frac{2p}{1+p^2}\left(\frac{z'}{2px^2} + \frac{3s^2x^2}{2ay(1+p^2)}\right) + 3\ldots$$

$$z'\left[\frac{1}{x^2(1+p^2)} + 3(z+1)\right] + \ldots$$

$$z'\left[1+3(z+1)x^2(1+p^2)\right] + \ldots$$

$$-\frac{4a^2\tau''y^4}{y} = z^2x^2y^2\left[3s^2c^2-s^4\right]\left[\frac{z'c}{sx^2}+\frac{3s^2x}{ay}z\right] + s^3cy^2\left[-z^2z'+4x^2zz'\right] - x^2z^2z'$$

$$= \frac{3s^2xz}{ay}z^2x^2y^2\left[3s^2c^2-s^4\right] + z'\left[z^2y^2\left[3sc^3-s^3c\right] - z^2y^2s^3c + 4x^2zy^2s^3c - x^2z^2\right]$$

$$A = \frac{2\,8a^6y^3}{27x^3(s^2-2y^2c^2)^3}\frac{-s^5}{4a^2y^4}\left\{\frac{3s^4x^3yz^3}{a}(3c^2-s^2) + z'\left[z^2y^2(3sc^3-2s^3c)+4x^2zy^2s^3c-x^2z^2\right]\right\} = \frac{-2}{3(s^2+2y^2c^2)^3}\left\{3s\ldots\right.$$

3

$$z'\left[2 + \frac{3}{4}(z^2-1)(p^2+1)\right] = -\frac{6p^3x^3z}{ay(1+p^2)}$$

$$z' = -\frac{6p^3x^3z}{ay(1+p^2)\left[2+3(z^2-1)(p^2+1)\right]}$$

$$[\cdot] = 2 + (3z^2-3)(-3z^2-2)$$

$$-9z^4 - 9z^2 + 3$$

$$\frac{p^4(2p^2+3)(y^2-x^2)^2}{(p^2+2y^2)^2}$$

$$\frac{3}{2}\frac{s^2 z}{ay}(y^2-x^2)$$

$$y' = -\frac{z'}{4xy}$$

$$p' + 3z' + 3zz' = 0$$

$$\left(\frac{z'}{px^2} + \frac{3p^3x^3z}{ay(1+p^2)}\right) + 3z' + 3zz' = 0$$

$$\left[\frac{1}{x^2(1+p^2)} + 3(z+1)\right] + \frac{3p^3xz}{ay(1+p^2)^2} = 0$$

$$\left[1 + 3(z+1)x^2(1+p^2)\right] + \frac{3p^3x^3z}{ay(1+p^2)} = 0$$

$$\frac{-2}{3(s^2+2y^2c^2)^3}\left\{3s^4 z^3(3c^2-s^2) + \frac{az'}{x^3y}\left[z^2y^2(3sc^3-2s^3c) + 4x^2zy^3s^3c - x^2z^2\right]\right\}$$

$$xy = \frac{z}{1+z^2} = \frac{-\frac{1}{4}}{1+\frac{1}{16}} = -\frac{4}{17}$$

$$z' = -4xy\,\psi' = \frac{16}{17}\psi'$$

$$p^2 + 3z + 3z^2 = 0 \qquad p = \frac{3}{c} = \frac{3}{4}$$

$$2\varphi' + 4\psi'\frac{4}{3}\frac{y}{x} + 3\frac{9}{25}\frac{x}{ay}\frac{1}{4} = 0$$

$$2pp' + 3z' + 6zz' = 0$$

$$\frac{3}{2}p' + \left(3 - \frac{3}{2}\right)z' = 0 \qquad p' = z'$$

$$\varphi' = -\frac{8}{3}\frac{y}{x}\psi'$$

$$\frac{25}{16}\varphi' = z'$$

$$\left(-\frac{128xy}{25} + \frac{16}{3}\frac{y}{x}\right)\psi' + \frac{27}{25a}\frac{x}{4y} = 0 \quad \cdot\frac{x}{y}$$

$$\frac{25}{16}\varphi' = -4xy\,\psi'$$

$$\varphi' = -\frac{64xy}{25}\psi'$$

$$\left(-\frac{128}{25}\cdot\frac{5}{8} + \frac{16}{3}\right)\psi' + \frac{27}{4\cdot 25a}\frac{5}{3} = 0 \quad \cdot\frac{15}{16}$$

$$\varphi' = +\frac{64xy}{25y}\frac{27\cdot 3}{2a\cdot 8} = \frac{81x}{400ay}$$

$$(-3+5)\psi' + \frac{27}{64a} = 0$$

$$p' = \varphi'\frac{25}{16} = \frac{81x}{16^2 ay}$$

$$\psi' = -\frac{27}{128a}$$

$$s = \frac{3}{5} \quad 18$$

$$-\frac{2p}{2p^2+} \qquad c = \frac{4}{5} \quad 48$$

$$z' = \frac{27xy}{32a} = \frac{27x}{32ay}\frac{3}{8} = \frac{81x}{16^2 ay}$$

$$\frac{x^2}{y^2} = \frac{5}{3} \qquad \frac{y^2}{x^2} = \frac{3}{5}$$

$$\ln \varsigma'' = -\ \ + \ln s^4 + \ln x^4 + \ln z^2 + \ln(2s^2+3c^2) - \ln c^3 - \ln y^2$$

$$\frac{\varsigma'''}{\varsigma''} = \frac{4c}{s}\varphi' + \frac{4y}{x}\psi' + \frac{2z'}{z} + \frac{-2sc}{2s^2+3c^2}\varphi' + \frac{3s}{c}\varphi' + \frac{2x}{y}\psi'$$

$$= \frac{4\cdot 4}{3}\left(\frac{81x}{400ay}\right) - \frac{4x}{y}\frac{27\cdot 3}{128a\cdot 5} + \frac{2\cdot 81x}{64ay} + \frac{-24}{66}\frac{81x}{400ay} + \frac{3\cdot 3}{4}\frac{81x}{400ay} - \frac{2x}{y}\frac{27}{128a}$$

$$= \frac{x}{ay}\left[\frac{27}{25} - \frac{81}{32\cdot 5} - \frac{81}{32} - \frac{81}{11\cdot 100} + \frac{9\cdot 81}{4\cdot 400} - \frac{27}{64}\right]$$

$$\ln \tau' = -\ln \varsigma^3 + \ln c + \ln x^2 + \ln z^2 - \ln y^2 \qquad p^2 = \frac{9}{16}$$

$$2y^2 = \frac{3}{4} = \frac{12}{16}$$

$$\frac{\tau''}{\tau'} = \frac{3c}{s}\varphi' + \frac{s}{c}\varphi' + \frac{2y}{x}\psi' + \frac{2z'}{z} + \frac{2x}{y}\psi'$$

7

II $10u^4 - 60z^3u^3 + [-84z^5 + 162z^4 - 24z^3 - 8z^2 - 8z + 12 - 60z^4 + 40z^3 + 50z^4 + 10z^3 - 30z^3 - 20z^2 - 20z - 30]u^2$

$+ [96z^5 - 208z^4 + 8z^3 - 18z^2 + 12z + 30z^2 - 20z]u + 30z^4 = 0$

~~$10u^4 - 60z^3$~~

10	u^4	$+5$	5
$-60z^3$	u^3	$-30z^3$	$\frac{30 \cdot 2^3}{3 \cdot 9} = \frac{80}{9}$
$-84z^5 + 162z^4 + 36z^3 + 12z^2 + 12z - 18$	u^2	$-42z^5 + 81z^4 + 18z^3 + 6z^2 + 6z - 9$	$\frac{475}{84}$
$96z^5 - 208z^4 + 8z^3 + 12z^2 - 8z$	u	$48z^5 - 104z^4 + 4z^3 + 6z^2 - 4z$	$-\frac{1840}{81}$
$+30z^4$	u^0	$15z^4$	$\frac{3.5.16}{81} = \frac{240}{81}$

$12z^5 - 16z^4 - 16z^3 + 24z^2 + 4z - 8$

-8 $+16$

-24

III $z^3[-30u^3 + (-42z^2 + 81z + 18)u^2 + (48z^2 - 104z + 4)u + 15z] + 5u^4 + (6z^2 + 6z - 9)u^2 + (6z^2 - 4z)u$

$-\frac{242}{3}$ 10

$= (u-1)\{z^3[-30u^2 + (-42z^2 + 81z - 12)u + (6z^2 - 23z - 8)] + 5u^3 + 5u^2 + (6z^2 + 6z - 4)u\} = 0$

$\frac{-56 + 162 - 24}{3}$ $= -2$ $\frac{8}{3}$ $\frac{+46 - 24}{3} = 10 = -15z$ $\frac{8 - 12 - 12}{3} = -\frac{16}{3}$ $\frac{8+3}{3} = \frac{11}{3}$

$\frac{II}{(u-1)} = 5u^3$

059777

-42 $+81$ $+18$ $+6$ $+6$ -9

$+28$

109

$-\frac{218 + 54}{3}$ $\frac{-164}{3}$ $\frac{328 + 54}{9} = \frac{382}{9}$ $\frac{-764 + 162}{27} = -\frac{602}{27}$ $\frac{1204 - 729}{81} = \frac{1324}{81}$ $\frac{475}{81}$

$\frac{II}{(u-1)} = 5u^3$

$\frac{-56+162-24}{3}$ $= -2$

$\frac{8}{3} \frac{+46-24}{3} = 10 = -157$

$\frac{8-12-12}{3} = -\frac{16}{3}$

$\frac{8+3}{3} = \frac{11}{3}$

059777

-42 +81 +18 +6 +6 -9

+28

109 $\frac{-218+54}{3}$ $\frac{-164}{3}$ $\frac{328+54}{9} = \frac{382}{9}$ $\frac{-764+162}{27} = -\frac{602}{27}$ $\frac{1204-729}{81} = \frac{1321}{81}$ $\frac{475}{81}$

48 -104 4 6 -4 0

-32

-136 $\frac{272+12}{3} = \frac{284}{3}$ $\frac{-568+54}{9} = \frac{-514}{9}$ $\frac{1028-108}{27} = \frac{920}{27}$ $-\frac{1840}{81}$

$81u^4 + 144u^3 + 95u^2 - 368u + 48 = 0$

225

320

$5u^4 + \frac{80}{9}u^3 + \frac{1425}{81}u^2 - \frac{1840}{81}u + \frac{240}{81} = 0$

880
720
240
1840!

~~$405u^4 + 720u^3 + 1321u^2 - 2176u + 240$~~

405
480
1321
240
815

$u^4 + \frac{16}{9}u^3 + \frac{95}{81}u^2 - \frac{368}{81}u + \frac{48}{81} = 0$ 5

a b c

$30\frac{8}{27}$

$-\frac{2a}{3} + b$

$\frac{N}{3}$ $\frac{-2a+3b}{3}$ $-\frac{8}{3}\left(\frac{-2a+3b}{3}\right) + c$

$-\frac{2}{3}\left(\frac{N}{9}\right) + c$

$\frac{42 \cdot 2^5}{3^5} + \frac{81 \cdot 2^4}{3^4} - \frac{18 \cdot 2^3}{3^3} + \frac{6 \cdot 2^2}{3^2} - \frac{6 \cdot 2}{3} - 9$

$+\frac{14 \cdot 32}{81} + 13 - \frac{8}{3} + \frac{8}{3}$ -15

9

II $(u-1)\left[81u^3 + 225u^2 + 320u - 48\right]$

I. $(u-1)\left[259u^3 - 87u^2 + 99u + 18\right]$

$\frac{14 \cdot 32}{81} + \frac{1}{3} =$

320
128
27
425!!!

-42 +81 +18 6 6 -9

+28

109 $\frac{-218+54}{3} = -\frac{164}{3}$ $\frac{328+54}{3} = \frac{382}{9}$ $\frac{-762+162}{27} = -\frac{602}{27}$ 1204-72

$\frac{1204-729}{81} = \frac{425}{81}$

I $259u^4 - 346u^3 + 186u^2 - 81u - 18 = 0$

$(u-1)\left[259u^3 - 87u^2 + 99u + 18\right]$

7

II $10u^4 - 60z^3u^3 + [-84z^5 + 162z^4 - 24z^3 - 8z^2 - 8z + 12 - 60z^4 + 40z^3 + 50z^4 + 10z^4 - 30z^3 + 20z^2 + 20z - 30]u^2$

$+ [96z^5 - 208z^4 + 8z^3 - 18z^2 + 12z + 30z^2 - 20z]u + 30z^4 = 0$

$10u^4 - 60z^3$

10	u^4		$+5$	5
$-60z^3$	u^3	$-30z^3$		$\frac{30 \cdot 2^3}{3 \cdot 9} = \frac{80}{9}$
$-84z^5 + 162z^4 + 36z^3 + 12z^2 + 12z - 18$	u^2	$-42z^5 + 81z^4 + 18z^3 + 6z^2 + 6z - 9$		$\frac{475}{84}$
$96z^5 - 208z^4 + 8z^3 + 12z^2 - 8z$	u	$48z^5 - 104z^4 + 4z^3 + 6z^2 - 4z$		$-\frac{1840}{81}$
$+30z^4$	u^0	$15z^4$		$\frac{3 \cdot 5 \cdot 16}{81} = \frac{240}{81}$

$12z^5 - 16z^4 - 16z^3 + 24z^2 + 4z - 8$

-8, -24, $+16$

III $z^3 [-30u^3 + (-42z^2 + 81z + 18)u^2 + (48z^2 - 104z + 4)u + 15z] + 5u^4 + (6z^2 + 6z - 9)u^2 + (6z^2 - 4z)u$

$-\frac{242}{3}$ 10

$= (u-1)\{z^3[-30u^2 + (-42z^2 + 81z - 12)u + (6z^2 - 43z - 8)] + 5u^3 + 5u^2 + (6z^2 + 6z - 4)u\} = 0$

-168 $\frac{-56 + 162 - 24}{3}$ $= -2$ $\frac{8}{3}$ $\frac{+46 - 24}{3} = 10 = -15z$ $\frac{8 - 12 - 12}{3} = -\frac{16}{3}$ $\frac{8+3}{3} = \frac{11}{3}$

$\frac{II}{(u-1)} = 5u^3$

059777

-42 $+81$ $+18$ $+6$ $+6$ -9

$+28$

109

$\frac{-218 + 54}{3}$ $\frac{-164}{3}$ $\frac{328 + 54}{9} = \frac{382}{9}$ $\frac{-764 + 162}{27} = \frac{-602}{27}$ $\frac{1904 - 729}{81} = \frac{1321}{81}$ $\frac{475}{81}$

$\times 81$

I

42   −81   12   4   4   −6

-28 / 109

$\frac{-218+36}{3} = \frac{-182}{3}$ $\frac{364+36}{9} = \frac{400}{9}$ $\frac{-800+108}{27} = \frac{-692}{27}$ $\frac{+1384-486}{81} = \frac{898}{81}$

−48   104   −4   24   −6   0

32 / 136

$\frac{-272-12}{3} = \frac{-284}{3}$ $\frac{+568+216}{9} = \frac{784}{9}$ $\frac{-1568-162}{27} = -\frac{1730}{27}$ $+\frac{3460}{81}$

$-15\frac{96}{81} = \frac{45\cdot 4}{9}$ $-\frac{240+1620}{81} = -\frac{1860}{81}$

180
1620

$\frac{5\cdot 4}{9} = \frac{20}{9} = \frac{180}{81}$

42   −81   12   4   4   −6

-28 / -109

$\frac{218+36}{3} = \frac{254}{3}$ $\frac{-508+36}{9} = \frac{-472}{9}$ $\frac{944+108}{27} = \frac{1052}{27}$ $\frac{-2104-486}{81} = \frac{-2590}{81}$

−2590	x^4	4^4
	3460	4^3
−1860	811	4^2
	810	4
	180	4^0
−4450	4450	

039778

$\delta''' =$

$\delta'' = \frac{110\cdot 9^3}{4^{10}\, 9^3}$ $2'' =$

$\int 45\, x_1^2 y$ $2\cdot 5\cdot 3$

$$\rho = \frac{x^2}{ac}$$

$$p^2 = -3z(1+z)$$

$$\frac{\rho'}{\rho} = \frac{3s^3xz}{2acy} = \frac{3.27x(-1)\,5}{125\,y\,(4)\,2a^4} = -\frac{81x}{800y}$$

$$z = -\tfrac{1}{4} \qquad p^2 = +\tfrac{3}{4}$$

$$x^2 = \frac{5}{8} \qquad s^2 = \frac{9}{25} \qquad c^2 = \frac{16}{25}$$

$$y^2 = \frac{3}{8} \qquad s = \frac{3}{5} \qquad c = \frac{4}{5}$$

$$-\frac{81}{800}\cdot\frac{25}{32}$$

32

$$\rho' = \frac{3s^3x^3}{2a^2c^2y}(y^2-x^2)$$

$$\tau = \frac{3x}{2ay}\left[s^2+2_y{}^2c^2\right]$$

$$\rho = \frac{5.5}{a\,8.4} = \frac{25}{32a}$$

$$\frac{\rho''}{\rho} = \frac{9s^6x^2z^2}{2a^2c^2y^2} + \frac{27s^4x^2z^2}{4a^2y^2} = \frac{9s^4x^2z^2}{4a^2c^2y^2}(2s^2+3c^2)$$

$$\rho' = \frac{3.27.5(-1)\,25}{125.8.4.2a^2.16}\frac{x}{y} = \frac{-81x}{1024a^2y}$$

18 48 66

$$\rho'' = \frac{9s^4x^4z^2}{4a^3c^3y^2}(2s^2+3c^2)$$

$$\rho'' = \frac{9.81.625.66.125.8}{625\;8^2.16.25.4a^3.66.8} = \frac{110.9^3}{8^5a^3}$$

22 5

$$\tau' = -\frac{9s^3cx^2z^2}{2a^2y^2}$$

$$ac\left[-2\frac{3s^3x^3z}{2a^2c^2y}\;\frac{9x^2}{4a^2y^2}(s^2+2y^2c^2)^2 + \frac{x^2}{ac}\;\frac{3x(s^2+2y^2c^2)}{2ay}\;\frac{9s^3cx^2z^2}{2a^2y^2}\right] + as\left[\frac{9s^4x^2z^2(2s^2+3c^2)}{4a^3c^3y^2}\;\frac{3x(s^2+2y^2c^2)}{}\right]$$

$$\tau = \frac{3x}{2ay}\left[\frac{9}{25}+\frac{2.3}{8}\cdot\frac{16}{25}\right] = \frac{3^2.7x}{2.25ay}$$

$$\rho = \frac{25}{32a} \qquad \tau = \frac{9.7x}{2.25ay}$$

$$\tau' = +\frac{9.27.4.8.(-1).8}{125.5.8.4.2a^2\,3} = \frac{9^2}{2.5^3}$$

$$\rho' = \frac{-9^2x}{4^5a^2y} \qquad \tau' = \frac{9^2}{2.5^3}$$

$$\rho'' = \frac{110.9^3}{4^{10}a^3} \qquad \tau'' =$$

$$\rho''' =$$

$$\left[s^3c(2\rho\rho'\tau + 3\rho^2\tau') + s^2c^2(3\rho\rho'' + \rho^2\tau^2 + 4\rho^4 - 4\rho'^2) + 4\rho\rho'\tau sc^3 \right] = a\rho s^2 c \left[s^2[2\rho\rho'\tau + 3\rho^2\tau' - \rho^2\tau^2] + sc(3\rho\rho'' + \rho^2\tau^2 + 4\rho^4 - 4\rho'^2 - 2\rho\rho'\tau) + c^2(4\rho\rho'\tau - \rho'^2) \right]$$

$-s^4\rho^2\tau^2 \quad -2\rho\rho'\tau \quad -\rho^2\tau^2 \quad -\rho'^2 \quad -2\rho\rho'\tau \quad -\rho'^2c^4$

$+\rho^2\tau^2 \quad -3\rho^2\tau' \quad +5\rho'^2 - 3\rho\rho'' \quad -2\rho\rho'\tau \quad +\rho'^2 \quad -9\rho^4s^4c^2$

$$9\rho^4 s^2 c^3 = a\rho s^2 c \left[s^2[\quad] + sc(\quad) + c^2(\quad) \right]$$

$$\underline{9\rho^3 c} = a \left[p^2(2\rho\rho'\tau + 3\rho^2\tau' - \rho^2\tau^2) + p(3\rho\rho'' + \rho^2\tau^2 + 4\rho^4 - 4\rho'^2 + 2\rho\rho'\tau) + 4\rho\rho'\tau - \rho'^2 \right]$$

$A + B = C \quad B_1 \qquad B = C - A$

$A_1 + B_1 = C_1 \quad -B \qquad B_1 = C_1 - A_1$

$B(C_1 - A_1) = B_1(C - A)$

$BC_1 - BA_1 - B_1C + B_1A = 0$

c

$cB - sB_1 = 9\rho^3(sc - s^3c) = 9\rho^3c^3$

$9\rho^4s^2c^2 + [s\rho\rho'\tau +]^2 = 9\rho^4s^4c^2 + s^3c \cdots$

$$a\rho\left\{ 9\rho^4 s^3 c \left[s^2\rho^2\tau^2 + 2\rho\rho'\tau sc + \rho'^2c^2 \right] - 9\rho^4 s^2c^2 \left[s^2(2\rho\rho'\tau + 3\rho^2\tau') + sc(3\rho\rho'' + \rho^2\tau^2 + 4\rho^4 - 4\rho'^2) + c^2 4\rho\rho'\tau \right] \right\} =$$

$$= 9\rho^4 s\left[s^2\rho^2\tau^2 + 2\rho\rho'\tau sc + \rho'^2c^2 \right] - 9\rho^4 s^2 c \left[s^2(2\rho\rho'\tau + 3\rho^2\tau') + \cdots \right]$$

$$9\rho^3 s(1 - a\rho s^2c) = a\left[s^2(2\rho\rho'\tau + 3\rho^2\tau') + sc(\quad) + 4\rho\rho'\tau c^2 \right]$$

$a\, 9\rho^4 s^3 c + aL = 9\rho^3 s \qquad -sc \qquad 9\rho^3 s(1 - a\rho s^2 c) = aL$

$a\, 9\rho^4 s^2c^2 + aM = 9\rho^3 s^2 c \qquad 9\rho^3 s^2 c(1 - a\rho c) = aM$

$9\rho^4(s^2c^2 - s^4c^2) + M - scL = 0 \qquad scL - M = 9\rho^4 s^2 c^4 \qquad c/L = s^2\left[2\rho\rho'\tau + 3\rho^2\tau'\right) + sc(3\rho\rho'' - 4\rho'^2 + \rho^2\tau^2 + 4\rho^4) + 4\rho\rho'\tau c^2$

$$L\, sc(1 - a\rho c) = M(1 - a\rho s^2 c)$$

$-s/\ M = s^2\rho^2\tau^2 + 2\rho\rho'\tau sc + \rho'^2c^2$

$scL - a\rho sc^2L = M - a\rho s^2cM$

$~~scL~~ \cdot 9\rho^4s^2c^4 = a\rho(sc^2L - s^2cM)$

$9\rho^3 sc^3 = a\rho(cL - sM)$

$a\, 9\rho^4 s^3 c + a L = 9\rho^3 s$ $\quad -sc \quad$ $9\rho^3 s(1 - a\rho s^2 c) = aL$

$a\, 9\rho^4 s^2 c^2 + aM = 9\rho^3 s^2 c$ $\quad$ $9\rho^3 s^2 c(1 - a\rho c) = aM$

$9\rho^4(s^2c^2 - s^4c^2) + M - scL = 0$ $\quad$ $scL - M = 9\rho^4 s^2 c^4$ $\quad$ $c/ L = s^2[2\rho\rho'\tau + 3\rho^2\tau') + sc(3\rho\rho'' - 4\rho'^2 + \rho^2\tau^2 + 9\rho^4) + 4\rho\rho'\tau c^2$

$$L\, sc(1 - a\rho c) = M(1 - a\rho s^2 c)$$

$-s/ \; M = \rho^2\rho^2\tau^2 + 2\rho\rho'\tau sc - \rho'^2 c^2$

$$scL - a\rho sc^2 L = M - a\rho s^2 c M$$

$$~~scL~~ \cdot 9\rho^4 s^2 c^4 = a\rho(sc^2L - s^2cM)$$

$$9\rho^3 sc^3 = a\rho(cL - sM)$$

$$9\rho^3 sc^3 = a\left[-s^3\rho^2\tau^2 + s^2c(3\rho^2\tau' + ~~2\rho\rho'\tau - 2\rho\rho'\tau~~) + sc^2(3\rho\rho'' - 5\rho'^2 + \rho^2\tau^2 + 9\rho^4) + 4\rho\rho'\tau c^3\right]$$

$$9\rho^3 s = a\left[-p^3\rho^2\tau^2 + 3p^2\rho^2\tau' + p(3\rho\rho'' - 5\rho'^2 + \rho^2\tau^2 + 9\rho^4) + 4\rho\rho'\tau\right]$$

$$9\rho^3 \underline{s}(2p + p^3) = a[p^3(2\rho\rho'\tau + 3\rho^2\tau')$$

$$~~\rho^3~~\; 81\rho^6 \frac{p^2(2+p^2)}{1+p^2} = a^2[p^7 - \quad]$$

$$a\left(9\rho^4p^2 + (\rho\tau p + \rho')^2(1+p^2)\right) = 9\rho^3\frac{p^2}{c} = a\left[\rho^2\tau^2p^4 + 2\rho\rho'\tau p^3 + (\rho'^2 + \rho^2\tau^2 + 9\rho^4)p^2 + 2\rho\rho'\tau p + \rho'^2\right]$$

$$a\left[-p^3\rho^2\tau^2 + p^2(3\rho^2\tau') + p(3\rho\rho'' + \rho^2\tau^2 + 9\rho^4 - 5\rho'^2) + 4\rho\rho'\tau\right] = 9\rho^3 s$$

$$a^2(\qquad)(\qquad) = 81\rho^6 p^3$$

$$-\rho^4\tau^4 p^7 + (3\rho^4\tau^2\tau'$$

(4) $\quad 4a^4(9\rho^4+\rho'^2)^3 - 9a^2\rho^2\{8\cdot 9^2\rho^8 - 180\rho^4\rho'^2 - \rho'^4\} + 9\cdot 18^2\rho^8 = 0$

et $\rho' = 3t\rho^2$ dans (4) →

$$4a^4\rho^4(t^2+1)^3 - a^2\rho^2\{8-20t^2-t^4\} + 4 = 0$$

$\Delta = t^2(t^2-8)^3 \qquad t = \dfrac{2+\alpha^2}{\alpha} \qquad t^2-8 = \dfrac{(\alpha^2-2)^2}{\alpha^2} \qquad t^2+1 = \dfrac{\alpha^4+5\alpha^2+4}{\alpha^2} = \dfrac{(\alpha^2+1)(\alpha^2+4)}{\alpha^2}$

$$\Delta = \frac{(\alpha^2+2)^2(\alpha^2-2)^6}{\alpha^8}$$

$$a^2\rho^2 = \frac{8 - 20\dfrac{(\alpha^2+2)^2}{\alpha^2} - \dfrac{(\alpha^2+2)^4}{\alpha^4} \pm \dfrac{(\alpha^2+2)^2(\alpha^2-2)^3}{\alpha^4}}{8\dfrac{(\alpha^2+1)^3(\alpha^2+4)^3}{\alpha^6}} = -\alpha^2\frac{+\alpha^8+28\alpha^6+96\alpha^4+112\alpha^2+16 \pm (\alpha^8-4\alpha^6+16\alpha^2-16)}{8(\alpha^2+1)^3(\alpha^2+4)^3}$$

$a^2\rho^2 = \dfrac{-4\alpha^2}{(\alpha^2+4)^3} \qquad a^2\rho^2 = \dfrac{-\alpha^4}{4(\alpha^2+1)^3}$ — se transforment l'une dans l'autre par substit. $\alpha = \dfrac{2}{\alpha'}$

Gardons 2e valeur, mettons $\alpha = \dfrac{2\nu}{1-\nu^2} \qquad \alpha^2+1 = \dfrac{(1+\nu^2)^2}{[1-\nu^2]^2} \qquad a^2\rho^2 = -\dfrac{16\nu^4}{(1-\nu^2)^4}\,\dfrac{(1-\nu^2)^6}{4(1+\nu^2)^6} = -\dfrac{4\nu^4(\nu^2-1)^2}{(\nu^2+1)^6}$

$ia\rho = \dfrac{2\nu^2(\nu^2-1)}{(\nu^2+1)^3} \qquad \dfrac{\rho'}{\rho} = 2\left[\dfrac{1}{\nu} + \dfrac{\nu}{\nu^2-1} - \dfrac{3\nu}{\nu^2+1}\right]\nu' \qquad \rho' = 3t\rho^2 \qquad \alpha^2+2 = \dfrac{2(1+\nu^4)}{(1-\nu^2)^2} \qquad t = \dfrac{2(1+\nu^4)}{(1-\nu^2)^2}\,\dfrac{1-\nu^2}{2\nu}$

$\dfrac{\rho'}{\rho} = 3t\rho = 3\dfrac{1+\nu^4}{(1-\nu^2)\nu}\,\dfrac{1}{ia}\,\dfrac{2\nu^2(\nu^2-1)}{(\nu^2+1)^3} = +\dfrac{3i}{a}\,\dfrac{2\nu^3(\nu^4+1)}{(\nu^2+1)^3}$

059779

$2\nu'\dfrac{-\nu^4+4\nu^2-1}{\nu(\nu^2-1)(\nu^2+1)} = \dfrac{3i}{a}\,\dfrac{2\nu^3(\nu^4+1)}{(\nu^2+1)^3} \qquad$ ~~$\nu^2 = p \mid p'\dfrac{-p^2+4p-1}{p(p-1)(p+1)} = \dfrac{3i}{a}\,\dfrac{2p(p^2+1)}{(p+1)^3}$~~

~~$\dfrac{dp(p+1)^2(-p^2+4p-1)}{p^2(p-1)(p^2+1)} = \dfrac{6i}{a}$~~ $\qquad \dfrac{\nu'(-\nu^4+4\nu^2-1)(\nu^2+1)^2}{\nu^2(\nu^2-1)(\nu^4+1)} = \dfrac{3i}{a}$

$\nu^2 = p \qquad \dfrac{(-p^2+4p-1)(p+1)^2}{p(p-1)(p^2+1)} = -1 + \dfrac{1}{p} + \dfrac{4}{p-1} + \dfrac{0}{p^2+1} \qquad \dfrac{-4p+4}{p^2+1}$

$-p^4$

$\alpha = \dfrac{2\nu}{1-\nu^2}$

$$-\nu' + \frac{\nu'}{\nu^2} + \frac{4\nu'}{\nu^2-1} + \frac{4\nu'(-\nu^2+1)}{\nu^4+1} = 3\frac{i}{a}$$

$t = \dfrac{1+\nu^4}{\nu(1-\nu^2)}$

$$-\nu - \frac{1}{\nu} - \ln(\nu+1)^2 + \ln(\nu-1)^2 + \sqrt{2}\ln\frac{\nu^2+\sqrt{2}\nu+1}{\nu^2-\sqrt{2}\nu+1} = 3\frac{i}{a}(s-s_0)$$

$ia\rho = \dfrac{2\nu^2(\nu^2-1)}{(\nu^2+1)^3} \qquad \nu' = \dfrac{3i}{a}\,\dfrac{\nu^2(\nu^2-1)(\nu^4+1)}{(-\nu^4+4\nu^2-1)(\nu^2+1)^2} \qquad \rho' = \dfrac{12}{a^2}\,\dfrac{\nu^3(\nu^4+1)(\nu^2-1)}{(\nu^2+1)^6}$

~~Vérification~~.

$$* \quad \frac{d\nu(\nu^2+1)^2(-\nu^4+4\nu^2-1)}{\nu^2(\nu^2-1)(\nu^4+1)} = \frac{3i}{a}ds$$

$$+\frac{1}{\nu} + 2\ln\frac{\nu-1}{\nu+1} + \sqrt{2}\ln\frac{\nu^2+\sqrt{2}\nu+1}{\nu^2-\sqrt{2}\nu+1} = \frac{3i}{a}(s-s_0)$$

D'autre part $dy = \rho\,ds$

$$-3i\,dy = 2\frac{-\nu^4+4\nu^2-1}{(\nu^2+1)(\nu^4+1)}d\nu$$

$$-3(y-y_0) = 2\int \frac{-3d\nu}{\nu^2+1} + \frac{2(\nu^2+1)}{\nu^4+1}d\nu = 2\left[-3\,\mathrm{arctg}\,\nu + \sqrt{2}\left\{\mathrm{arctg}\,\sqrt{2}\left(\nu+\tfrac{\sqrt{2}}{2}\right) + \mathrm{arctg}\,\sqrt{2}\left(\nu-\tfrac{\sqrt{2}}{2}\right)\right\}\right]$$

$$= 2\left[-3\,\mathrm{arctg}\,\nu + \sqrt{2}\,\mathrm{arctg}\,\frac{\sqrt{2}\nu}{1-\nu^2}\right]$$

$\mathrm{arctg}\,x = \dfrac{1}{2i}\ln\left(\dfrac{1+ix}{1-ix}\right)$

$$a = \frac{\;\dots\;}{\rho'^2 \cos^2 + 9\rho''^2 \sin^2\cos^2} = \frac{\;\dots\;}{(\rho'^2 + 9\rho''^2\sin^2)\cos}$$

$$(5\rho'^2 - 3\rho\rho'')\sin^2 + \rho'^2\cos^2 - 9\rho''\sin^4 = 0$$

(1) $9\rho''\sin^4 + (-4\rho'^2 + 3\rho\rho'')\sin^2 - \rho'^2 = 0$

(2) $81\rho^8 a^2 s^6 + (81\rho^6 - 81\rho^3 a^2 + 18\rho'^2\rho^4 a^2)s^4 + (\rho'^4 - 18\rho'^2\rho^4)a^2 s^2 - a^2\rho'^4 = 0$

En posant $y = 9\rho'' - 5\rho'^2 + 3\rho\rho''$

on obtient $9\rho''[9\rho^2 - a^2 y]s^2 - \rho'^2 y a^2 = 0$ $\qquad s^2 = \dfrac{a^2\rho'^2 y}{9\rho''[9\rho^2 - a^2 y]}$

(1) donne

$$9\rho'' a^4 \rho'^4 y^2 + 9a^2\rho^4\rho'^4 [9\rho^2 - a^2 y][y - 9\rho'' + \rho'^2] - 81\rho^8\rho'^4[9\rho^2 - a^2 y]^2 = 0$$

(3) $-a^4 y^3 + 9a^2\rho^2 y^2 + 9a^2\rho^2 y(9\rho'' + \rho'^2) - 9^3\rho^8 = 0$

Pour que rac. double, il faut si $at^3 + bt^2 + ct + d = 0$, $\quad 4(3ac - b^2)^3 + (27a^2d - 9abc + 2b^3)^2 = 0$

$$4\{-3\cdot 9a^6\rho^2(9\rho'' + \rho'^2) - 81a^4\rho^4\}^3 + \{-27a^8 9^3\rho^8 + 9^3 a^6\rho^4(9\rho'' + \rho'^2) + 2\cdot 9^3 a^6\rho^6\}^2 = 0$$

$$-4\cdot 27^3 a^{12}\rho^6\{a^2(9\rho'' + \rho'^2) + 3\rho^2\}^3 + 9^6 a^{12}\rho^8\{-27a^2\rho^4 + a^2(9\rho'' + \rho'^2) + 2\rho^2\}^2 = 0$$

$$-4\cdot 27\{a^6(9\rho'' + \rho'^2)^3 + 9a^4\rho^2(9\rho'' + \rho'^2)^2 + 27a^2\rho^4(9\rho'' + \rho'^2) + 27\rho^6\} + 27\rho^2\{a^4(+81\rho^8 - 36\rho^4\rho'^2 + \rho'^4) + a^2(-72\rho^6 + 4\rho^4\rho'^2) + 4\rho^4\}$$

$$-4a^4(9\rho'' + \rho'^2)^3 + 9a^2\rho^2[-4(9\rho'' + \rho'^2)^2 + 3(81\rho^8 - 36\rho^4\rho'^2 + \rho'^4)] + 108\rho^4\{-27\rho(9\rho'' + \rho'^2) - 18\rho^4 + \rho^4\} = 0$$

$$-4a^4(9\rho'' + \rho'^2)^3 + 9a^2\rho^2[8\cdot 9^2\rho^8 - 180\rho^4\rho'^2 - \rho'^4] - 4\cdot 27^2\rho^8 = 0$$

Relat. entre dernières d'ellipse et courbure

$$\left(\frac{a^2b^2}{\rho^2}\right)^{\frac{2}{3}} - (a^2 + b^2)\left(\frac{a^2b^2}{\rho^2}\right)^{\frac{1}{3}} + a^2b^2 + \frac{\rho'^2 a^2 b^2}{9\rho^4} = 0$$

059780

(5) $\rho^2 z^3 - a^2b^2 = 0$

(6) $9\rho^4 z^2 - 9\rho^4(a^2 + b^2)z + a^2b^2(9\rho^4 + \rho'^2) = 0$

$-9\rho^4 \qquad\qquad + 9\rho^2 a^2 b^2$

$$-9\rho^4(a^2 + b^2)z^2 + a^2b^2(9\rho^4 + \rho'^2)z + 9\rho^2 a^2 b^2 = 0$$

$$a^2b^2[81\rho^6 + 9\rho^4(9\rho^4 + \rho'^2)(a^2 + b^2)]^2 + [9\rho^4(9\rho^4 + \rho'^2)a^2b^2 - 81\rho^8(a^2 + b^2)^2][81\rho^6 a^2b^2(a^2 + b^2) + (9\rho^4 + \rho'^2)^2 a^2b^2] = 0$$

$$a^2b^2[9\rho^8 + (9\rho^4 + \rho'^2)(a^2 + b^2)]^2 + 9\rho^4[(9\rho^4 + \rho'^2)a^2b^2 - 9\rho^4(a^2 + b^2)^2][81\rho^6(a^2 + b^2) + a^2b^2(9\rho^4 + \rho'^2)^2] = 0$$

$b^4[9\rho^4(9\rho^4 + \rho'^2) \qquad\qquad 2a^2b^2[a^2 9\rho^4 (.$

On a (5) $\rho^2 z^3 - a^2 h^2 = 0$

(6) $9\rho^4 z^2 - 9\rho^4(a^2+h^2)z + a^2h^2(9\rho^4+\rho'^2) = 0$

Il faut avoir rac. double en h^2, donc $\frac{D(5,6)}{D(z,h^2)} = 0$

$$\begin{vmatrix} 3\rho^2 z^2 & -a^2 \\ 18\rho^4 z - 9\rho^4(a^2+h^2) & -9\rho^4 z + a^2(9\rho^4+\rho'^2) \end{vmatrix} = 0$$

Diviser par $3\rho^2$, il reste

(7) $-9\rho^4 z^3 + a^2 z^2(9\rho^4+\rho'^2) + 6a^2\rho^2 z - 3a^2\rho^2(a^2+h^2) = 0$

$+9\rho^2(5)$ $9\rho^4 z^3$ $-3a^2\rho^2(\ \ 3h^2) = 0$

$\cdot\frac{1}{a^2}$ (8) $z^2(9\rho^4+\rho'^2) + 6\rho^2 z - 3\rho^2(a^2+4h^2) = 0$

$\frac{1}{\rho^2}[6 + (5)(9\rho^4+\rho'^2)]$ (9) $z^2(9\rho^4+\rho'^2) + 9\rho^2 z - 3\rho^2(3a^2+3h^2) = 0$

(9)-(8) $3\rho^2 z = 3\rho^2(2a^2 - h^2)$ $z = 2a^2 - h^2$ $h^2 = 2a^2 - z$

(5) → (5') $\rho^2 z^3 + a^2 z - 2a^4 = 0$ | $\cdot(9\rho^4+\rho'^2)$

(8) $z^2(9\rho^4+\rho'^2) + 18\rho^2 z - 27a^2\rho^2 = 0$ | $\rho^2 z$

(9) $18\rho^4 z^2 - z a^2[27\rho^4 + (9\rho^4+\rho'^2)] + 2a^4(9\rho^4+\rho'^2) = 0$

L'élimination de z^2 entre (8) et 9 donne

$$4a^8(9\rho^4+\rho'^2)^4 + 4\cdot18\cdot27\, a^6\rho^6(9\rho^4+\rho'^2)^2 + 18^2 27^2 a^4\rho^{12} = \{18^2\rho^6 + a^2(9\rho^4+\rho'^2)(27\rho^4+9\rho^4+\rho'^2)\}$$

$$\{27a^4\rho^2[27\rho^4+9\rho^4+\rho'^2] - 36a^4\rho^2(9\rho^4+\rho'^2)\}$$

$$4a^4(9\rho^4+\rho'^2)^3 + 4\cdot18\cdot27\, a^2\rho^6(9\rho^4+\rho'^2) = 18^2\cdot27\rho^8 - 18^2\,36\rho^8 + \rho^2a^2(36\rho^4+\rho'^2)\,9\{27\rho^4-\rho'^2\}$$

(4) $4a^4(9\rho^4+\rho'^2)^3 - 9a^2\rho^2\{8\cdot9^2\rho^8 - 180\rho^4\rho'^2 - \rho'^4\} + 9\cdot18^2\rho^8 = 0$

c. à. d. la relation (4). Donc effectivement (4) donne enveloppe singulière.

Encore question à étudier : est-ce que (4) donne solution de 3 ?

Soit $\rho^2 = x$ $\rho' = y$ $\rho\rho'' = 2xyy'$ $y = 9x^2 - 5y^2 + xyy'$

$y = 3tx$ $y' = 3t'x + 3t$ $y = 9x^2(1+t^2+6xtt')$ $\rho'^2 = 9t^2x^2$

(4) → $4a^4x^2(1+t^2)^3 - a^2x\{8 - 20t^2 - t^4\} + 4 = 0$ Soit $t^2 = u$

$4a^4x^2(1+u)^3 + a^2x\{-8+20u+u^2\} + 4 = 0$ $y = 9x^2(1+u+3xu')$

$xu'\{12a^4x^2(1+u)^2 + a^2x(20+2u)\} + 8a^4x(1+u)^3 + a^2\{-8+20u+u^2\} = 0$ $1+u = p$

$(3) \to \; a^4x^2(p+3xp')^3 - a^2x(p+3xp')^2 - a^2xp(p+3xp') + 1 = 0$

$(4')\quad p'\{12a^4x^2p^2 + a^2x(2p+18)\} + 8a^4xp^3 + a^2\{p^2+18p-27\} = 0$

$$p+3xp' = \frac{-12a^2xp^3 + \alpha^2(-p^2-36p+81)}{12a^2xp^2 + 2p+18} = \frac{(p-9)^2}{12a^2xp^2+2p+18} - p$$

$(10)\quad a^4x^2[-12a^2xp^3+(-p^2-36p+81)]^3 - a^2x[-12a^2xp^3+(-p^2-36p+81)]^2[12a^2xp^2+2p+18] - a^2xp[-12a^2xp^3+(-p^2-36p+81)][12a^2xp^2+2p+18]^2$

$+(12a^2xp^2+2p+18)^3 = 0$

(4) ri'solu ... in a^2x: $\Delta = (p-1)(p-9)^3$

$$a^2x = \frac{-p^2-18p+27 + (p-9)\sqrt{(p-1)(p-9)}}{8p^3}$$

si $p=9$, $a^2x = -\frac{1}{27}$ $\quad + \frac{1}{9^2}\left[\frac{12}{27}\cdot 27 + 4\cdot 9^2\right]^3 + \frac{1}{27}$ $\quad p+3xp' = -p \quad -a^4x^2p^3 - a^2xp^2 + a^2xp^2 + 1 = 0$

$-\frac{1}{27^2}9^3+1=0$

$p+3xp' = \alpha - p$

$a^4x^2(\alpha^3 - 3\alpha^2p + 3\alpha p^2 - p^3) - a^2x(\alpha^2 - 2\alpha p + p^2) - a^2xp\alpha + a^2xp^2 + 1 = 0$

$\alpha^3a^4x^2 - \alpha^2(3a^4x^2p + a^2x) + \alpha(3a^4x^2p^2 + a^2xp) - a^4x^2p^3 + 1 = 0$

$\frac{1}{4}(p-9)^{\frac{3}{2}}\{\sqrt{p-9} + 3\sqrt{p-1}\}$

$$\alpha = \frac{(p-9)^2}{2[6a^2xp^2+p+9]}$$

$(p-9)^6a^4x^2p^3 - (p-9)^4\,2(6a^2xp^3+p^2+9p)(3a^4x^2p^3+a^2xp^2) + (p-9)^2p^4(6a^2xp^3+p^2+9p)^2 + (1-a^4x^2p^3)\,8(6a^2xp^3+p^2+9p)^3 = 0$

si $p=0$ $\quad -27x^2x+4=0 \quad \alpha = \frac{81}{18} = \frac{9}{2} \quad a^2x = \frac{4}{27} \quad -27$

$9^6\frac{4^2}{9^8} - 9^4\,2\,3\,8\,\frac{4}{27} + 9^8\,4\,9^2 + 8\cdot 9^3 = 0$

$16 - 24 \ldots$

$$-8a^4x^2p^3 + 8 = \frac{-(p-1)(p-9)^3 + (p^2+18p-27)\sqrt{(p-1)(p-9)}}{4p^3}$$

$6a^2xp^3 + p^2 + 9p = \frac{1}{4}(p-9)^{\frac{3}{2}}\{\sqrt{p-9} + 3\sqrt{p-1}\}$

$(p-9)^{\frac{3}{2}}a^4x^2p^3 - (p-9)^4\,2\,\frac{1}{4}(p-9)^{\frac{3}{2}}\{\sqrt{p-9}+3\sqrt{p-1}\}[3a^4x^2p^3 + a^2xp^2] + (p-9)^{\frac{1}{2}}\,4p\,\frac{1}{16}(p-9)^3\{\ \}^2 +$

$+ (1-a^4x^2p^3)\,8\,\frac{1}{64}(p-9)^{\frac{3}{2}}\{\ \}^3$

$a^4x^2p^3\left\{(p-9)^{\frac{3}{2}} - (p-9)\frac{3}{2}\{\sqrt{p-9}+3\sqrt{p-1}\} - \frac{1}{8}\{\sqrt{p-9}+3\sqrt{p-1}\}^3\right]$

$-a^2xp^2\,\frac{p-9}{2}\{\sqrt{p-9}+3\sqrt{p-1}\} + (p-9)^{\frac{1}{2}}\frac{p}{4}\{\sqrt{\ }+3\sqrt{\ }\}^2 + \frac{1}{8}\{\sqrt{\ }+3\sqrt{\ }\}^3$

$$p+3xp' = p\,\frac{-2p+6+3\sqrt{(p-1)(p-9)}}{p+3}$$

$$4a^4x^2p^3\left[\frac{-2p+6+3\sqrt{\ }}{p+3}\right]^3 - 4a^2xp^2\frac{(-2p+6+3\sqrt{\ })^2}{(p+3)^2} - 4a^2xp^2\,\frac{-2p+6+3\sqrt{\ }}{p+3} + 4 = 0$$

$$\frac{-4a^2xp^2}{(p+3)^2}(-2p+6+3\sqrt{\ })(-p+9+3\sqrt{\ })$$

$a^2x(-2p+6+3\sqrt{\ })\{(-p^2-18p+27)(-2p+6+3\sqrt{\ })^2 + 4p^2(p+3)(+p-9-3\sqrt{\ })\} + 4(p+3)^3 - 4(-2p+6+3\sqrt{\ })^3 = 0$

$8(p-1)(-p^3-12p^2+225p-351) + 3p(4p^2-27p+27)\sqrt{\ } + 3p[p-1-\sqrt{\ }][4p^2-36p+48+(-3p+15)\sqrt{\ }]$

$[5p^3-27p^2+135p-81+(-5p^2-30+27)\sqrt{\ }]\{(p-1)(-p^3-12p^2+225p-351) + 4(4p^2-27p+27)\sqrt{\ }\} + 32p^3[p-1-\sqrt{\ }][4p^2-36p+48+(-3p+15)\sqrt{\ }]$

$(p-1)\{-p^4+26p^3+504p-621+(-p^3-p^2+112p-243)\sqrt{\ }\} \quad (p-1)\cdot 8$

059784

$-\sin\varphi \qquad \cos\varphi \qquad 0$

$0 \qquad 0 \qquad 1$

$x' \;(\; 0 \qquad a\varphi' \qquad f' \;)$

$x'' \;(\; -a\varphi'^2 \qquad a\varphi'' \qquad f'' \;)$

$x''' \;(\; -3a\varphi'\varphi'' \qquad a(\varphi''' - \varphi'^3) \qquad f''' \;)$

$a\varphi' = \cos u$

$f' = \sin u$

$\varphi' = \frac{\cos u}{a} \qquad \varphi'' = -\frac{\sin u}{a}u'$

$f' = \sin u \qquad \varphi''' = -\frac{\sin u}{a}u'' - \frac{\cos u}{a}u'^2$

$\rho^2 = a^2\varphi'^4 + a^2\varphi''^2 + f''^2$

$$\rho^2\tau = \begin{vmatrix} 0 & a\varphi' & f' \\ -a\varphi'^2 & a\varphi'' & f'' \\ -3a\varphi'\varphi'' & a(\varphi''' - \varphi'^3) & f''' \end{vmatrix} = a^2\left[f'(\varphi'^5 - \varphi'^2\varphi''' + 3\varphi'\varphi''^2) + f''(-3\varphi'^2\varphi'') + f'''\varphi'^3\right]$$

$$= a^2\varphi'\left[f'(\varphi'^4 - \varphi'\varphi''' + 3\varphi''^2) - 3\varphi'\varphi''f'' + \varphi'^2 f'''\right]$$

$x' \;(\; 0 \qquad \cos u \qquad \sin u$

$x'' \;(\; -\frac{\cos^2 u}{a} \qquad -\sin u\, u' \qquad \cos u\, u'$

$x''' \;(\; 3\frac{\cos u \sin u}{a}u' \qquad -\sin u\, u'' - \cos u\, u'^2 - \frac{\cos^3 u}{a^2} \qquad \cos u\, u'' - \sin u\, u'^2$

$\rho^2 = u'^2 + \frac{\cos^4 u}{a^2} \qquad u'^2 = \rho^2 - \frac{\cos^4}{a^2}$

$\cos u \qquad \sin u$

$-\sin u \qquad \cos u$

$x' \;(\; 1 \qquad 0 \qquad 0 \;)$

$x'' \quad 0 \qquad u' \qquad -\frac{\cos^2 u}{a}$

$x''' \;(\; -u'^2 - \frac{\cos^4 u}{a^2} \qquad u'' + \frac{\sin u \cos^3}{a^2} \qquad 3\frac{\sin u \cos}{a}u'$

$\rho^2\tau = 3\frac{\sin u \cos}{a}u'^2 + u''\frac{\cos^2}{a} - \frac{\sin u \cos^5}{a^3}$

$\frac{\tau^2}{\rho^2} =$

$\frac{1}{4}\left[1 + \frac{4}{u-1} + \frac{1}{u+1}\right]$

$u + \ln\frac{(u-1)^4}{u} = \rho_1^4$

$-\sin\varphi$ $\cos\varphi$

ρ 0

x' (0 ay'

x'' ($-ay'^2$ ay''

x''' ($-3ay'y''$ $ay''' - y'^3$

$\rho^2 = a^2y'^4 + a^2 y$

$\rho^2\tau = \begin{vmatrix} 0 & ay' \\ -ay'^2 & ay'' \\ -3ay'y'' & a(y''' - y'^3) \end{vmatrix}$

x' (0 $\cos$

x'' ($-\frac{\cos^2 u}{a}$ $-\sin u$

x''' ($3\frac{\cos u \sin u}{a}u'$ $-\sin u$

$\cos$

$-\sin$

x' (1 1

x'' 0 u'

x''' ($-u'^2 - \frac{\cos^4 u}{a^2}$ $u'' + \sin$

Cyl

$\varphi = A \frac{\sqrt{s^2+1}}{s}$ $\quad \tau = B + \frac{1}{1+s^2}$ $= \frac{B(1+s^2)+1}{1+s^2}$ $\quad 3A^2 = B^2 + 2B\,?$

$R = \frac{1}{A}\frac{s}{\sqrt{s^2+1}}$

$R^2 + (R'T)^2 = a^2$

$R' = \frac{1}{A}\frac{1}{\sqrt{\ }} - \frac{s^2}{\sqrt{\ }^3} = \frac{1}{A}\frac{1}{\sqrt{\ }^3}$

$\frac{s^2}{A^2(1+s^2)} + \frac{1}{A^2(1+s^2)^3}\frac{(1+s^2)^2}{[B(1+s^2)+1]^2} = a^2$

$x = a\cos\varphi$

$y = a\sin\varphi$

$z = f$

Courbe cylindrique.

$X' = (-a\sin\varphi\,\varphi',\ a\cos\varphi\,\varphi'\ \ f')$ $\quad a^2\varphi'^2 + f'^2 = 1$

$X'' = (-a\sin\varphi\,\varphi'' - a\cos\varphi\,\varphi'^2,\ a\cos\varphi\,\varphi'' - a\sin\varphi\,\varphi'^2,\ f'')$

$X''' = (-a\sin\varphi\,\varphi''' - a\cos\varphi\,\varphi'\varphi'' - 2a\cos\varphi\,\varphi'\varphi'' + a\sin\varphi\,\varphi'^3$

$a\cos\varphi\,\varphi'' - 3a\sin\varphi\,\varphi'\varphi'' - a\cos\varphi\,\varphi'^3 \quad f''')$

$\cos\varphi$	$\sin\varphi$	0
$-\sin\varphi$	$\cos\varphi$	0
0	0	1

059784

$X'\ (0 \quad a\varphi' \quad f')$

$X''\ (-a\varphi'^2 \quad a\varphi'' \quad f'')$

$X'''\ (-3a\varphi'\varphi'' \quad a(\varphi''' - \varphi'^3) \quad f''')$

$a\varphi' = \cos u$

$f' = \sin u$

$\varphi' = \frac{\cos u}{a} \quad \varphi'' = -\frac{\sin u}{a}u'$

$\rho^2 = a^2\varphi'^4 + a^2\varphi''^2 + f''^2$

$f' = \sin u \quad \varphi''' = -\frac{\sin u}{a}u'' - \frac{\cos u}{a}u'^2$

ρ

$\frac{\sqrt{s^2+1}}{s}$

$n' = -\rho t + \tau b$

$n'' = -\rho' t - (\rho^2+\tau^2) n + \tau' b$

$\left(\frac{1-t^2}{2t}\right)' = \frac{-4t^2 - 1 + t^2}{2t^2}$

$n'^2 = \rho^2 + \tau^2$

$s^2 + 1 = s^2 + 2ts + t^2$

$n''^2 = (\rho^2+\tau^2)^2 + \rho'^2 + \tau'^2$

$s = \frac{1-t^2}{2t}$ $\quad ds = -\frac{1}{2}$

$n' n'' = \rho\rho' + \tau\tau'$

$n'^2 n''^2 - (n' n'')^2 = \frac{(\rho^2+\tau^2)^4 + (\rho^2\tau' - \tau\rho')^2}{}$ $\quad (s^2+1) = (s+t)^2 = \left(\frac{1+t^2}{2t}\right)^2$

$\rho^2 + \tau^2 + \frac{(\rho\tau' - \tau\rho')^2}{(\rho^2+\tau^2)^3}$

$\frac{1+t^2}{2t} \; \frac{2t}{t^2-1} \; \frac{t^2+1}{2t} \, dt$

$t' = \rho n$ $\quad s' = \rho$ $\quad t_1 = n$

$\frac{1}{4} \frac{(1+u)^2}{(u-1)u} du$

$t'' = -\rho^2 t + \rho' n + \rho\tau b$ $\quad n_1 s' \rho_1 = -\rho t + \tau b$

$\rho^2 \rho_1^2 = \rho^2 + \tau^2$

$u^2 - u + 4u - [illegible]$

$\tau = \frac{B(1+s^2) + 1}{(1+s^2)}$ $\quad \rho_1^2 = 1 + \frac{\tau^2}{\rho^2}$

$u^2 + 2u + 1$

$\frac{\tau^2}{\rho^2} =$

$\frac{1}{4}\left[1 + \frac{4}{u-1} - \frac{1}{u+1}\right]$

$u + \ln \frac{(u-1)^4}{u} = s_1$

$\rho = A \frac{\sqrt{s^2+1}}{s}$ $\quad \tau = B$ [illegible]

$R^2 + (R'T)^2 = a^2$

$\frac{s^2}{A^2(1+s^2)} + \frac{1}{A^2(1+s^2)^3}$ $\quad \frac{(1+s^2)}{[B(1+s}$ [illegible]

$x = a \cos \varphi$

$y = a \sin \varphi$

$z = f$

$X' = (-a \sin\varphi\, \varphi', \; a\cos\varphi\, \varphi'$ [illegible]

$X'' = (-a\sin\varphi\,\varphi'' - a\cos\varphi\,\varphi'^2, \; a$ [illegible]

$X''' = (-a\sin\varphi\,\varphi''' - a\cos\varphi\,\varphi'\varphi'' -$ [illegible]

$a\cos\varphi\,\varphi'' - 3a\sin\varphi\,\varphi'$ [illegible]

$\cos\varphi$	$\sin\varphi$	0
$-\sin\varphi$	$\cos\varphi$	0
0	0	1

X'	(0	$a\varphi'$
X''	($-a\varphi'^2$	$a\varphi''$
X'''	($-3a\varphi'\varphi''$	$a(\varphi''' - \varphi'^3)$

$\rho^2 = a^2\varphi'^4 + a^2\varphi''^2 +$ [illegible]

$$(t^2+1)\left\{\frac{1}{R^2T^2}t^4+3\frac{T'}{R^2T^2}t^3+\left(5\frac{R'^2}{R^4}+3\frac{R''}{R^3}-6\frac{R'^2}{R^4}\right)t^2+2\frac{R'}{R^3T}t+\frac{R'^2}{R^4}\right\}-\frac{9}{R^4}t^4=0$$

$$\rho' = -\frac{R'}{R^2}$$

$$\rho'' = -\frac{R''}{R^2}+\frac{2R'^2}{R^3}$$

$$(t^2+1)\left\{R^2t^4+3R^2T't^3+T^2(3RR''-R'^2)t^2+2R^2TR't+T^2R'^2\right\}-9T^2t^4=0$$

$$T=fR$$

$$(t^2+1)\left\{R^2t^4+3R^2(f'R+fR')t^3+f^2R^2(3RR''-R'^2)t^2+2fR^2R't+f^2R^2R'^2\right\}-9f^2R^2t^4=0$$

$$f'=0 \qquad (t^2+1)\left\{t^4+3fR't^3+f^2(3RR''-R'^2)t^2+2fR't+f^2R'^2\right\}-9f^2t^4=0$$

$$t^6+3fR't^5+t^4(1+f^2$$

$fR'=x$

~~$R'=y$~~

$f^2(3RR''-R'^2)=z$

$9f^2=y$

$$t^6 ~~+3fR't^5~~ + (1+t)t^4+3$$

$$(t^2+1)\left\{t^4+3xt^3+zt^2+2xt+x^2\right\}-y\,t^4=0$$

$\rho''=(\rho\tau t)=A$

$\tau'=(\rho\tau t)=B$

$\rho'=(\rho\tau t)=C$

$\rho''=C'$ — 4 eq, $\rho\rho'\rho''\tau\tau'$ 5 par; A_ρ, +3, +2

$D=C'_\rho C+C_\tau B+C_t t'-A=0$

$D_\rho C+D_\tau B+D_t t'+C_t t''=0$

$$(t^2+1)\left\{t^4-3at^3+ht^2-2ct+c^2\right\}-d^2t^4=0$$

039782

$$\rho=\frac{d\tau}{3}$$

$$3\rho'=\frac{d\tau^2c}{3}=\frac{d'\tau+d\tau'}{3}$$

$$dc\tau^2=d'\tau+\frac{da\tau^2}{3}$$

$$\tau'=\frac{d\tau^2}{3}$$

$$d(3c-a)\tau=d'$$

$$\tau=\frac{d'}{d(3c-a)} \qquad \rho=\frac{d'}{3(3c-a)}$$

$$t^6-3at^5+(1+h-d^2)t^4-(2c+3a)t^3+(h+c^2)t^2-2ct+c^2=0$$

$$6t^5-15at^4+4(1+h-d^2)t^3-3(2c+3a)t^2+2(h+c^2)t-2c=0$$

$$-3at^5+2(1+h-d^2)t^4-3(2c+3a)t^3+4(h+c^2)t^2-10ct+6c^2=0$$

$$30t^4-60at^3+12(1+h-d^2)t^2-6(2c+3a)t+2(h+c^2)=0$$

$$-15at^4+8(1+h-d^2)t^3-9(2c+3a)t^2+8(h+c^2)t-10c=0$$

$$2(1+h-d^2)t^4-6(2c+3a)t^3+12(h+c^2)t^2-40ct+30c^2=0$$

$$(1+h-d^2)t^4-(6c+9a)t^3+(6h+6c^2)t^2-20ct+15c^2$$

~~$5t^4+2(1+h-d^2)t^3 \ldots -2ct$~~

$(t^2+1)\{t^4-3\alpha t^3+ht^2-2ct+c^2\}-d^2t^4=0$

039782

$\rho = \frac{d\tau}{3}$

$t^6-3\alpha t^5+(1+h-d^2)t^4-(2c+3\alpha)t^3+(h+c^2)t^2-2ct+c^2=0$

$3\rho' = \frac{d\tau^2 c}{3} = \frac{d'\tau+d\tau'}{3}$

$dc\tau^2 = d'\tau + \frac{d\alpha\tau^2}{3}$

$6t^5-15\alpha t^4+4(1+h-d^2)t^3-3(2c+3\alpha)t^2+2(h+c^2)t-2c=0$

$-3\alpha t^5+2(1+h-d^2)t^4-3(2c+3\alpha)t^3+4(h+c^2)t^2-10ct+6c^2=0$

$\tau' = \frac{d\tau^2}{3}$

$d(3c-\alpha)\tau = d'$

$30t^4-60\alpha t^3+12(1+h-d^2)t^2-6(2c+3\alpha)t+2(h+c^2)=0$

$-15\alpha t^4+8(1+h-d^2)t^3-9(2c+3\alpha)t^2+8(h+c^2)t-10c=0$

$\tau = \frac{d'}{d(3c-\alpha)}$ $\quad \rho = \frac{d'}{3(3c-\alpha)}$

$2(1+h-d^2)t^4-6(2c+3\alpha)t^3+12(h+c^2)t^2-40ct+30c^2=0$

$(1+h-d^2)t^4-(6c+9\alpha)t^3+(6h+6c^2)t^2-20ct+15c^2$

~~$5t^4+2(1+h-d^2)t^2-2ct$~~

$\frac{1}{\tau} = \frac{d(3c-\alpha)}{d'}$

$\left[\frac{d'}{3(3c-\alpha)}\right]' = c\,\frac{d'^2}{3d(3c-\alpha)^2}$

$15t^4+6(1+h-d^2)t^2-6ct+h+c^2 = \alpha(30t^3+9t)$

$-3\left(\frac{d(3c-\alpha)}{d'}\right)' = \alpha$

$\alpha = \frac{15t^4+6(1+h-d^2)t^2-6ct+h+c^2}{30t^3+9t}$

$\left(\frac{-5t^3-9t}{-15t^4-27t^2}\right)\frac{15t^4+6(1+h-d^2)t^2-6ct+h+c^2}{3t(10t^2+3)}$

$+8(1+h-d^2)t^3-18ct^2+8(h+c^2)t-10c=0$

$\alpha(-30t^3-9t)+h(6t^2+1)+d^2(-6t^2)+15t^4+6t^2-6ct+c^2=0$ $\quad -t^2 \mid 8t$

$\alpha(-15t^4-27t^2)+h(8t^3+8t)+d^2(-8t^3)+8t^3-18ct^2+8c^2t-10c=0$ $\quad -7t$

$\alpha(-18t^3)+h(t^4+6t^2)+d^2(-t^4)+t^4-6ct^3+6c^2t^2-20ct+15c^2=0$ $\quad 6 \mid 8$

$\alpha(30t^5-45t^3)+h(35t^2)$ $\quad -15t^6+6ct^3-c^2t^2-36ct^3+36c^2t^2-120ct+90c^2=0$

$\alpha(6t^5-9t^3)+h\,7t^2-3t^6-6ct^3+7c^2t^2-24ct+18c^2=0$

$-6\alpha-8-\beta=0$ $\quad 6\alpha+8\gamma+\beta=0$ $\quad \alpha=\beta=8$

$\gamma=-7$

$6\alpha+8+\beta=0$ $\quad \alpha+8\gamma+6\beta=0$

$\alpha+8+6\beta=0$

$\alpha(-240t^5-72t^3+105t^5+189t^3-72t^3)+h(48t^3+8t^2-56t^4-56t^2+8t^4+48t^2)+d(-48t^2$

Cyl rac. triple.

$t = \operatorname{tg} \varphi$, on a relations

$$\begin{cases} 3\rho'^2 - 2\rho\rho'\tau t + \rho^2\tau^2 t^4 + 9\rho^4 t^4 \dfrac{3t^2-1}{(t^2+1)^3} = 0 \\ 9\rho\rho'' t - 2\rho\rho'\tau(5t^2-3) + \rho^2\tau^2 t^3(5t^2+6) + 18\rho^4 \dfrac{t^3(9t^4+2t^2-3)}{(t^2+1)^3} = 0 \\ -\rho^2\tau' t^2 + 2\rho\rho'\tau + 8\rho^2\tau^2 t^3 - 72\rho^4 \dfrac{t^2}{(t^2+1)^3} = 0 \end{cases}$$

Equation en t : $\rho^2\tau^2 \sin^4\varphi - 3\rho^2\tau' \sin^3 \cos + (5\rho'^2 - 3\rho\rho'') \sin^2\cos^2 - 2\rho\rho'\tau \sin\cos^3 + \rho'^2\cos^4 - 9\rho^4 \sin^4 \cos^2 = 0$

$\dfrac{3\tau'}{\tau^2} = a$ $\quad \dfrac{5\rho'^2 - 3\rho\rho''}{\rho^2\tau^2} = h$ $\quad \dfrac{3\rho}{\tau} = d$ $\quad \dfrac{\rho'}{\rho\tau} = c$

~~Conoïde~~ ~~cylindrique~~

$$(t^2+1)\left\{t^4 - at^3 + ht^2 - 2ct + c^2\right\} - d^2 t^4 = 0$$

$$\rho^2 = u'^2 + \frac{\cos^4}{a^2} u \qquad \rho^2 \tau = 3\frac{\sin\cos}{a} u'^2 + u'' \frac{\cos^2}{a} - \frac{\sin\cos^5}{a^3}$$

ρ' u, u', u''

ρ'' u, u', u'', u'''

$$\rho^2 = u'^2 + \frac{\cos^4 u}{a^2}$$

$$\rho\rho' = u'u'' - \frac{2\cos^3\sin}{a^2} u'$$

$$\rho'^2 + \rho\rho'' = u''^2 + u'u''' - 2\frac{\cos^3\sin}{a^2} u'' - \frac{2(\cos^4 - 3\cos^2\sin^2)}{a^2} u'^2$$

$$2\rho\rho'\tau + \rho^2\tau' = 3\frac{\cos^2-\sin^2}{a} u'^3 + 6\frac{\sin\cos}{a} u'u'' + \frac{\cos^2}{a} u''' - \frac{2\cos\sin}{a} u'u'' - \frac{(\cos^6 - 5\cos^4\sin^2)}{a^3} u'$$

$u' = x \quad u'' = y \quad u''' = z$

~~$$\rho^2\tau = y\frac{c^2}{a} + 3x^2\frac{cs}{a} - \frac{c^5 s}{a^3}$$~~

$$\rho^2 = x^2 + \frac{c^4}{a^2} \qquad \text{~~}y = \frac{\rho\rho'}{x} + 2\frac{c^3 s}{a^2}\text{~~}$$

~~$$\rho\rho' = xy - 2\frac{c^3 s}{a^2} x$$~~

039783

~~$$\rho'^2 + \rho\rho'' = zx + y^2 - 2y\frac{c^3 s}{a^2} - 2x^2\frac{(c^4 - 3s^2 s^2)}{a^2}$$~~

$$x \quad 2\rho\rho'\tau + \rho^2\tau' = 3\, z\frac{c^2}{a} + 4xy\frac{sc}{a} + 3x^3\frac{c^2-s^2}{a} - x\frac{c^6 - 5c^4 s^2}{a^3}$$

$$-\frac{c^2}{a^2} \qquad \rho'^2 + \rho\rho'' = zx + \frac{\rho^2\rho'^2}{x^2} + 2\frac{\rho\rho' c^3 s}{a^2 x} - 2x^2\frac{(c^4 - 3c^2 s^2)}{a^2}$$

$$x(2\rho\rho'\tau + \rho^2\tau') - \frac{c^2}{a^2}(\rho'^2 + \rho\rho'') = 4x\frac{cs}{a}\left[\rho\rho' + 2x\frac{c^3 s}{a}\right] + 3x^4\frac{c^2-s^2}{a} + x^2\frac{c^6 + c^4 s^2}{a^3}$$

$$- \frac{\rho^2\rho'^2}{x^2}\frac{c^2}{a^2} + 2\frac{\rho\rho' c^5 s}{a^3 x} + 2x^2\frac{c^6 - 3c^4 s^2}{a^3}$$

$$x\,(2\rho\rho'\tau + \rho^2\tau') = 8\,x\frac{c^2}{a^2} + 4xy\frac{sc}{a} + 3x^3\frac{c^2-s^2}{a} - x\frac{c^6-5c^4s^2}{a^3}$$

$$-\frac{c^2}{a^2} \qquad \rho'^2+\rho\rho'' = 2x + \frac{\rho^2\rho'^2}{x^2} + 2\frac{\rho\rho' c^3 s}{a^2 x} - 2x^2\frac{c(c^4-3c^2s^2)}{a^2}$$

$$x(2\rho\rho'\tau+\rho^2\tau') - \frac{c^2}{a^2}(\rho'^2+\rho\rho'') = 4x\frac{cs}{a}\left[\rho\rho' + 2x\frac{c^3 s}{a}\right] + 3x^4\frac{c^2-s^2}{a} + x^2\frac{c^6+7c^4s^2}{a^3}$$

$$-\frac{\rho^2\rho'^2}{x^2}\frac{c^2}{a^2} + 2\frac{\rho\rho'c^5s}{a^3x} + 2x^2\frac{c^6-3c^4s^2}{a^3}$$

$$\rho^2\tau = \frac{\rho\rho'c^2}{ax} + 3x^2\frac{cs}{a} + \frac{c^5s}{a^3}$$

$$+3\rho^2\frac{cs}{a} - 2\frac{c^5s}{a^3}$$

$$\boxed{x^2 = \rho^2 - \frac{c^4}{a^2}}$$

$$\boxed{\frac{\rho\rho'c^2}{a\left(\rho^2\tau - 3\rho^2\frac{cs}{a} + 2\frac{c^5s}{a^3}\right)} = x}$$

$$x\left[2\rho\rho'\tau + \rho^2\tau' - 4\rho\rho'\frac{cs}{a}\right] - \frac{c^2}{a^2}(\rho'^2+\rho\rho'') =$$

$$= 3\left(\rho^4 - 6\rho^2\frac{c^4}{a^2} + 3\frac{c^8}{a^4}\right)\frac{c^2-s^2}{a} + \rho^2\frac{c^6+7c^4s^2}{a^3} - \frac{c^{10}+7c^8s^2}{a^5}$$

$$-\frac{\rho^2\rho'^2c^2}{x^2a} + 2\frac{\rho\rho'c^5s}{a^3x}$$

$$= 3\rho^4\frac{c^2-s^2}{a} + \frac{\rho^2}{a^3}\left(-6c^6+6c^4s^2+c^6+7c^4s^2 - 6c^4s^2\right) + \frac{c^8}{a^5}\left(3c^2-3s^2-c^2-7s^2+4s^2\right)$$

$$+2\frac{\rho\rho'c^3s}{a^3}\left(a\rho^2\tau - 3\rho^2cs + 2\frac{c^4s}{a^2}\right)$$

Déveloper au pt. où $z = -\frac{2}{3}$ $p^2 = \frac{2}{3}$ Soit $a =$ $s^2 = \frac{2}{5}$ $c^2 = \frac{3}{5}$

$a = \sqrt{5}$

Cylindre en $\xi \eta \zeta$: $(X - A)^2 =$ $ab = \sqrt{2}$ $ac = \sqrt{3}$

~~ξ - a cos y~~

$$\xi^2 + \eta^2 + \zeta^2 - 2a\xi\cos y - 2a\eta\sin y - [\xi\cos x - \eta\sin x \sin y + \zeta\sin x\cos y]^2 = 0$$

$$\xi^2\sin^2 x + \eta^2(1 - s^2x^2) + \zeta^2(1 - c^2x^2) + 2\xi\eta\, sxy + 2\eta\zeta\, sc x^2 - 2\xi\zeta\, cxy - 2\sqrt{2}\xi - 2\sqrt{3}\eta = 0$$

$y^2 - x^2 = -\frac{2}{3}$

$y^2 + x^2 = \frac{3}{3}$

0.9785

$\tau = \frac{3}{2}\frac{\sqrt{5}}{\sqrt{5}}\left[\frac{2}{5} + 2\frac{3}{5}\frac{1}{6}\right.$

$= \frac{3}{2}\frac{3}{5} = \frac{9}{10}$

$y^2 = \frac{1}{6}$ $x^2 = \frac{5}{6}$ $x^2y^2 = \frac{5}{36}$ $xy = \frac{\sqrt{5}}{6}$ $s = \frac{\sqrt{2}}{\sqrt{5}}$ $c = \frac{\sqrt{3}}{\sqrt{5}}$

$$\frac{5}{6}\xi^2 + \eta^2\left[1 - \frac{2}{5}\frac{5}{6}\right] + \zeta^2\left(1 - \frac{3}{5}\frac{5}{6}\right) + 2\xi\eta\frac{\sqrt{2}}{\sqrt{5}}\frac{\sqrt{5}}{6} + 2\eta\zeta\frac{\sqrt{2}}{\sqrt{5}}\frac{\sqrt{3}}{\sqrt{5}}\frac{5}{6} - 2\xi\zeta\frac{\sqrt{3}}{\sqrt{5}}\frac{\sqrt{5}}{6} - 2\sqrt{2}\xi - 2\sqrt{3}\eta = 0$$

$$5\xi^2 + 4\eta^2 + 3\zeta^2 + 2\sqrt{2}\xi\eta + 2\sqrt{6}\eta\zeta + 2\sqrt{3}\xi\zeta - 12\sqrt{2}\xi - 12\sqrt{3}\eta = 0$$

	s	$\frac{s^2}{2}$	$\frac{s^3}{3!}$	$\frac{s^4}{4!}$	$\frac{s^5}{5!}$	$\frac{s^6}{6!}$
ξ	1		$-\rho^2$	$-3\rho\rho'$	$-4\rho\rho'' - 3\rho'^2 + \rho^4 + \rho^2\tau^2$	
η		ρ	ρ'	$\rho'' - \rho^3 - \rho\tau^2$	$\rho''' - 6\rho^2\rho' - 3\rho'\tau^2 - 3\rho\tau\tau'$	
ζ			$\rho\tau$	$2\rho'\tau + \rho\tau'$	$3\rho''\tau + 3\rho'\tau' + \rho\tau'' - \rho^3\tau - \rho\tau^3$	

ξ^2

ξ 1 $\frac{25}{108}$ $s^2 + \frac{25}{3.108}s^4$

η $\rho = \frac{5\sqrt{3}}{18}$ $\rho' = \frac{3}{2}\frac{2\sqrt{3}}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{6}}\frac{5}{6\sqrt{3}}\left(-\frac{1}{3}\right)\frac{1}{\sqrt{5}} - \sqrt{6} = -\frac{2\sqrt{2}}{18}$

ζ $\frac{5\sqrt{3}}{18}\cdot\frac{9}{10} = \frac{\sqrt{3}}{4}$